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Probabilistic load-bearing capacity analysis of the slab bridge made of prestressed concrete girders

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Abstract: The paper presents a comprehensive procedure to determine a reliability and a load-bearing capacity level of the existing bridges on highways and roads. Advanced methods of reliability analysis based on simulation techniques of Monte Carlo type in combination with nonlinear finite element method analysis are used. The reliability index is considered as a main criterion of the reliability level of the existing structures as this index is described in current structural design standards, e.g. ISO and Eurocode. The target design value of the reliability index is possible to adjust in relation to the Bridge Management System and to the maintenance of the existing bridge structures, according to the system of main and control inspections of bridges. A single-span slab bridge made of precast prestressed concrete girders (60 years in operation) is used as example. Its load-bearing capacity is determined for the ultimate limit state and the remaining lifetime 2, 5, 10, 20, 30 and 40 years. The structural design load capacity was estimated by the fully probabilistic nonlinear FEM analysis using the simulation technique Latin Hypercube Sampling (LHS). Load-bearing capacity values based on a fully probabilistic analysis are compared with the load-bearing capacity levels estimated by deterministic methods of a critical section of the most loaded girders. The design resistance of the girder at the ultimate limit state is calculated by methods using a global safety factor according to the Eurocode 2 and the fib Model Code 2010 (the method of the Estimation Coefficient of Variation ECoV). The adjustment of the target design value of reliability index and partial safety factors of loads, materials and model resistance and load uncertainties was performed for each remaining lifetime in relation to the detailed diagnostic survey and in relation to main and control inspections.

1 Introduction

Bridges have to be designed in relation to the assumed reference period and the reliability index to resist all possible load combinations that might appear during theirs usage time. Assessment of reliability of existing bridge structures is the subject of research recently, e.g. [3], [4], [10] and [11]. It can be performed by deterministic approach, which requires a design action effect E_d o be smaller than a design resistance R_d of the structure or by stochastic approach, when calculated failure probability P_f is lower than the required failure probability $P_{f,t}$ for the given reference period.

An equivalent term to the failure probability is the reliability index β_t that is a commonly used measure of reliability of existing bridge structures. The reliability index β_t is formally defined in terms of the probability of failure $P_{f,t}$ as:

$$\beta_t = -\Phi^{-1}(P_{f,t}), \tag{1}$$

where $-\Phi^{-1}(P_{f,t})$ is the inverse function of the standardized normal probability distribution. It represents only an approximation, as Φ does not follow normal probability distribution function in general case.

Reliability index updating in relation to satisfactory inspection and maintenance policies of the existing bridge structures was described by Koteš & Vičan [6]. Their theoretical approach considers that the inspecting structure reaches a common level of the failure probability $P_{f,t}$ = 7,2.10^{-5} (β_t = 3,80). An inspection carried out in time $t_{insp} < T_d$ (T_d is a designed bridge working life, 100 years) will have a positive result that the verified bridge member should not fail in the sense of exceeding any of its limit states.

Another assumption is that that the structural resistance *R(t)* and the action effect *E(t)* are time dependent functions with a normal distribution. Positive result inspection can be described by following relation

$$\max(E_i) < R, for\ i = 1,2 \ldots N(t_{insp}), \tag{2}$$

where E_i are mutually independent load effects which occur in succession randomly in time.

The failure probability P_f of the observed structural element at the remaining lifetime period (t_{insp}, T_d) can be obtained by a conditional probability in accordance with

$$P_f = \frac{P_f(T_d) - P_f(t_{insp})}{1 - P_f(t_{insp})}, \tag{3}$$

where $P_f(T_d)$is the failure probability at the planned remaining lifetime and $P_f(t_{insp})$ is the failure probability at the inspection time of the bridge structure with updated information about the technical state of the structure and its load-bearing capacity. Results of the updated reliability index β_t in relation to the age of the bridge structure and its remaining lifetime are shown in Table 1.

Table 1: Reliability levels for existing bridge evaluation for bridges with age ≥ 60 years without degradation (according to [6])

Remaining lifetime [years]	The age of the bridge [years]							
	60 years		70 years		80 years		90 years	
	β_t	$P_{f,t}$	β_t	$P_{f,t}$	β_t	$P_{f,t}$	β_t	$P_{f,t}$
2	2,828	2,35.10^{-3}	2,777	2,78.10^{-3}	2,732	3,15.10^{-3}	2,692	3,56.10^{-3}
5	3,098	9,75.10^{-4}	3,053	1,13.10^{-3}	3,014	1,29.10^{-3}	2,978	1,45.10^{-3}
10	3,279	5,22.10^{-4}	3,239	6,00.10^{-4}	3,204	6,78.10^{-4}	3,172	7,57.10^{-4}
20	3,434	2,97.10^{-4}	3,401	3,45.10^{-4}	3,371	3,74.10^{-4}		
30	3,512	2,22.10^{-4}	3,483	2,48.10^{-4}				
40	3,561	1,85.10^{-4}						

2 Structural safety assessment using FEM nonlinear analysis

Methods for reliability and load-bearing capacity assessment using advanced techniques of reliability analysis have been described in Červenka [1] and Šomodíková et al. [13]. In the following text deterministic and stochastic approaches of the existing bridge structure reliability analysis, which use nonlinear FEM simulations at a structural element level and a whole structure level, according to the international standards for designing building structures will be introduced.

2.1 Deterministic approach

Bridge structure load-bearing capacity can be expressed for the dominant actions of load as

$$V_n = \frac{R_d - \gamma_g . \gamma_{Ed} . E_g}{\gamma_V . E_{v,n}} . \widetilde{V_n} \tag{4}$$

$\widetilde{V}_n$ is a traffic load value given e.g. in tons, R_d is a designed structural resistance, E_g are permanent load effects and E_v are single accidental traffic load effects. Adjustment of partial safety coefficients of material γ_m, model uncertainties γ_{Rd}, permanent load effects γ_g, traffic load effects γ_v and model uncertainties of load effects γ_{Ed} for desired reliability index β_t was described in Sýkora et al.[12].

2.1.1 Non-linear analysis according to EN1992 – part 2

Designed resistance is calculated according to EN1992 – part 2 as

$$R_d = \frac{r(\tilde{f}_{c,m}, \tilde{f}_{y,m}, \dots .)}{\tilde{\gamma}_R . \gamma_{Rd}}, \tag{5}$$

where $\tilde{\gamma}_R$ is a global safety factor of resistance, γ_{Rd} is a model uncertainty coefficient. Material characteristics $\tilde{f}_{c,m}, \tilde{f}_{y,m}, \dots$. are considered by their adjusted values.

2.1.2 ECOV method –variation coefficient estimation

This method proposed by Červenka [1] is based on the idea, that the random distribution of resistance, which is described by the coefficient of variation V_R, can be estimated from mean $\hat{R}_m$ and characteristic values $\hat{R}_k$. The underlying assumption is that random distribution of resistance is according to lognormal distribution, which is typical probabilistic model for structural resistance. In this case, it is possible to express the coefficient of variation as

$$V_R = \frac{1}{1{,}645} ln\left(\frac{\hat{R}_m}{\hat{R}_k}\right) \tag{6}$$

Global safety factor $\hat{\gamma}_R$ of resistance can be then estimated as

$$\hat{\gamma}_R = \exp(0{,}8 . \beta_t . V_R), \text{for } V_R \leq 0{,}2 \tag{8}$$

Eventually the design resistance R_d is calculated as a division of the mean resistance value $\hat{R}_m$ and a product of the global safety factor $\hat{\gamma}_R$ of resistance and model uncertainties γ_{Rd}.

2.2 Fully probabilistic approach

Probabilistic analysis of resistance and action can be performed by numerical method of Monte Carlo-type of sampling, such as LHS sampling method. Results of this analysis provide random parameters of resistance and actions, such as mean, standard deviation, etc. and the type of distribution function for resistance.

2.2.1 *Design resistance estimation according to EN 1990*

Value of the design resistance R_d is estimated by assuming two-parameter lognormal distribution of the probability as

$$R_d = R_m . \exp(-0{,}8 . \beta_t . V_r) \tag{9}$$

Mean resistance value R_m and coefficients of variation V_r are calculated according to the aggregate statistics of random simulations of structural response with a vector of material characteristics and model uncertainties

$$R = r(f_c, f_{ct}, E_c, f_{s,y}, f_{s,p}, \dots \theta_R) \tag{10}$$

2.2.2 *Design load-bearing capacity estimation according to EN 1990*

Load-bearing capacity function S is estimated according to the aggregate statistics of random simulations of structure response with a vector of random variables

$$S = s(f_c, f_{ct}, E_c, f_{s,y}, f_{s,p}, \theta_R \dots, g_0, g_1, V_i, \theta_E) \tag{11}$$

Material parameters, permanent loads g_0, g_1 and model uncertainties of resistance θ_R and uncertainties of model load effects θ_E are considered as random variables.

Random traffic load V_i is assumed to be deterministic. Design value of the load-bearing capacity $V_{i,d}$ is estimated as

$$P\left(S \leq V_{i,d}\right) = \Phi(-\beta_t) \tag{12}$$

3 Example of the load-bearing capacity estimation of the slab bridge

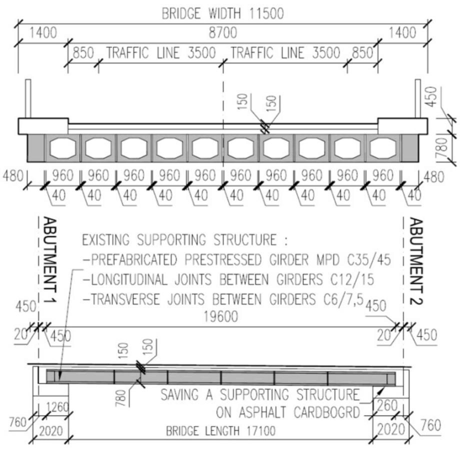

Figure 1: Side view of the analyzed bridge, transversal section and longitudinal section of the bridge.

A bridge with an ID 55-046 built in 1950 bridges a road class I over the railway. The bridge is considered as a single-slab span object with a prefabricated supporting structure and a monolithic substructure. Superstructure of the bridge is a simple beam consisting of 12 prefabricated and precast prestressed concrete girders MPD4 (10 intermediate) a MPD3 (2 outlying). Length of the superstructure is 20,50 m, length of the bridging is 17,50 m and width of the supporting structure is 11,0 m.

Detailed diagnostic survey of the superstructure did not show any damages of girders MPD4 a MPD3 as a result of exceeding neither the ultimate limit state nor the serviceability limit state. Corrosion of the reinforcement and prestressed tendons was not recorded either. Based on this positive result of the supporting structure inspection the estimation of its normal load-bearing capacity is performed for the reliability index according to Table 1.

3.1 Numerical model

3.1.1 Numerical model

By assumption of forming ideally rigid slab from individual girders MPD3 a MPD4 thanks to their transverse prestressing, plain FEM model was created using the computer program ATENA 2D [2]. Concrete parts of each girder were modeled as a material model 3D Non Linear Cementitious 2, prestressed reinforcement was modeled as a discrete member and soft reinforcement of the girder individual segments was modeled as smeared.
In both cases the reinforcement material was modeled by bi-linear working diagram with hardening. Simple beam placing on abutments was assumed. Structure model was loaded by its dead load, transverse prestressing effects and other permanent loads. Loading scheme for assessment of normal load-bearing capacity, which consists of continuous load and a two-axle vehicle (load model LM1), is presented. Loading was simulated by increasing of deformation or by loading force until a limit state was observed which corresponds to the loading level at limit state of decompression (D), the limit state of crack formation (T) and the limit load-bearing capacity (U), see Figure 2.

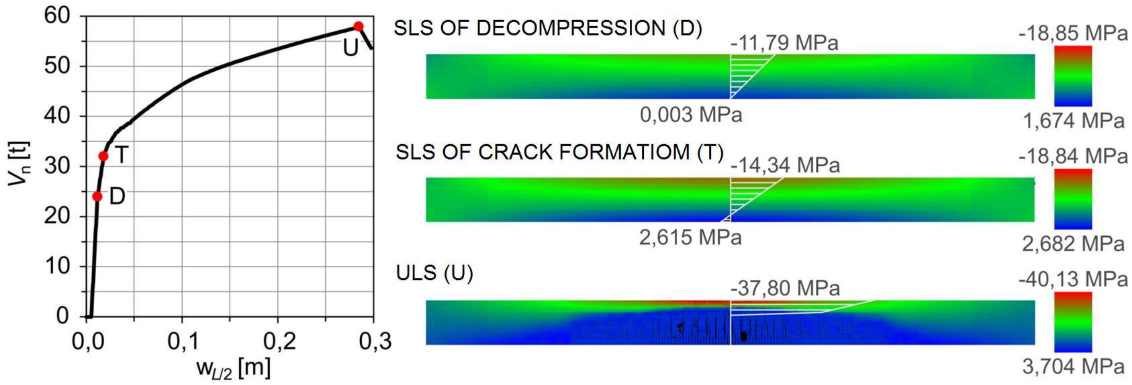

Figure 2: Diagram of normal load-bearing capacity – deflection; direct stress at the supporting structure.

3.2 Deterministic analysis

3.2.1 Load-bearing capacity of the superstructure at the structure member level

Load-bearing capacity of the bridge superstructure was estimated as the lowest value of the designed load-bearing capacities of the girders MPD4 a MPD3 for dominant actions of load according to Eq. (4). The design resistance of the girder was estimated based on two separate non-linear FEM analysis.

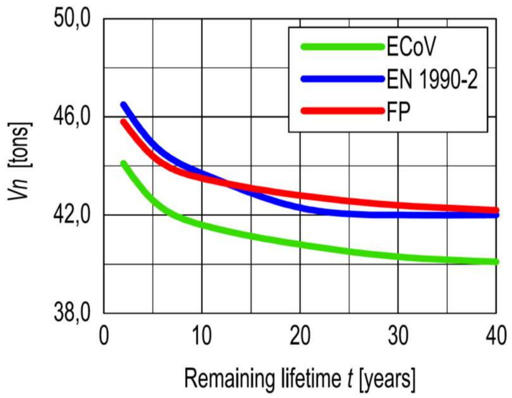

Figure 3: Normal Load-bearing capacity for ULS vs. remaining lifetime (Deterministic method).

Table 2: Normal load-bearing capacity for ULS vs. remaining lifetime (Deterministic method).

Probability index (Remaining lifetime)	Normal Load-bearing capacity V_n [tons]		
	EC2	ECoV	FP
β_t = *2,828* (2 years)	46,5	44,1	45,8
β_t = *3,098* (5 years)	44,9	42,6	44,4
β_t = *3,279* (10 years)	43,7	41,6	43,5
β_t = *3,434* (20 years)	42,3	40,8	42,8
β_t = *3,512* (30 years)	42,2	40,3	42,4
β_t = *3,561* (40 years)	42,0	40,1	42,2

Normal load-bearing capacity levels for the ultimate limit state (ULS) was estimated according to the required reliability index β_t and the remaining lifetime 2, 5, 10, 20, 30 a 40 years. The age of the superstructure is 60 years. Resulting values of the normal load-bearing capacity V_n are shown in Table 2. Figure 3 depicts the normal load-bearing capacity in relation to the remaining lifetime estimated according to the method in EN1992-2 (blue line), using the method of variation coefficient estimation - ECoV (green line) and based on the fully probabilistic analysis FP (red line).

3.3 Fully probabilistic analysis

3.3.1 Stochastic modeling of basic variables

Stochastic parameters of basic variables were defined using software FReET [8] according to Joint Committee on Structural Safety [5], including model uncertainties [9], and these were updated based on the material properties of concrete and reinforcement obtained from diagnostic survey.

Table 3: Definition of input parameters for assessment of load-bearing capacity.

Value		Unit	PDF*	Mean	CoV
Parameters of concrete girder MPD					
Young's modulus	E_c	[GPa]	LN 2 par.	37,2	0,10
Tensile strength	f_{ct}	[MPa]	LN 2 par.	3,30	0,15
Compressive strength	f_c	[MPa]	LN 2 par.	43,4	0,08
Fracture energy	G_f	[N/m]	LN 2 par.	$8{,}25.10^{-5}$	0,15
Mass density	γ_c	[kN/m^3]	N	23,80	0,03
Parameters of concrete transverse joints between girders					
Young's modulus	E_c	[GPa]	LN 2 par.	34,0	0,15
Tensile strength	f_{ct}	[MPa]	LN 2 par.	2,81	0,35
Compressive strength	f_c	[MPa]	Triang.	19,1	<29,8;8,5>
Fracture energy	G_f	[N/m]	LN 2 par.	$4{,}78.10^{-5}$	0,25
Mass density	γ_c	[kN/m^3]	N	23,80	0,08
Parameters of prestressed tendons					
Young's modulus	E_p	[GPa]	N	190,0	0,03
Ultimate strain	$\varepsilon_{p,lim}$	[-]	N	0,05	0,07
Yield strength	$f_{p.y}$	[MPa]	N	1248,0	0,03
Ultimate strength	$f_{p,u}$	[MPa]	N	1716,0	0,03
Parameters of reinforcement					
Young's modulus	E_s	[GPa]	N	200,0	0,07
Ultimate strain	$\varepsilon_{s,lim}$	[-]	N	0,05	0,07
Yield strength	$f_{s,y}$	[MPa]	LN 2 par.	462,1	0,07
Ultimate strength	$f_{s,u}$	[MPa]	LN 2 par.	581,4	0,07
Load and prestressing force					
Second dead load	g_1	[kN/m^2]	N	6,600	0,05
Load model LM1	V_n	[tons]	Det.	32,0	-
Force in time t = 60 years.	P_t	[MN]	N	$P_{m,t}$	0,09
Model uncertainties					
Resitance R	$\theta_{R,M}$	[-]	LN 2 par.	1,00	0,10
Load E	$\theta_{E,M}$	[-]	LN 2 par.	1,00	0,10

* N – normal distribution, LN 2 par. – 2-parametric lognormal distribution, Det. – deterministic value, Triang. – triangular distribution

In total 32 random simulations were generated using stratified Latin Hypercube Sampling (LHS) method, which is capable to cover space of random variables in terms of relatively small number of sample [7]. Material parameters of concrete as well as parameters of reinforcement, prestressed tendons and a value of secondary dead load were chosen as random variables. The self-weight of the structure and prestressed force were also randomized using concrete mass density. Definitions of random input variables by their probability density functions (PDF), mean values and coefficients of variation (CoV) are summarized in Table 3. Statistical correlation between concrete and reinforcement material parameters was also considered and imposed with respect to the formerly performed tests and JCSS recommendations using the simulated annealing approach [14]. Finally, the traffic load for determination of the load-bearing capacity was defined using a deterministic value of load according to the valid loading schemes introduced in current codes for design.

3.3.2 Probabilistic analysis of the load-bearing capacity

Design value of the load-bearing capacity of the superstructure of a bridge was estimated according to Eq. (12) considering the same reliability index β_t as in previous case of the deterministic analysis at the structure member level. Resulting values of normal load-bearing capacity V_n are summarized in Table 4. Figure 4 depicts the normal load-bearing capacity in relation to the remaining lifetime using the fully probabilistic method of analysis.

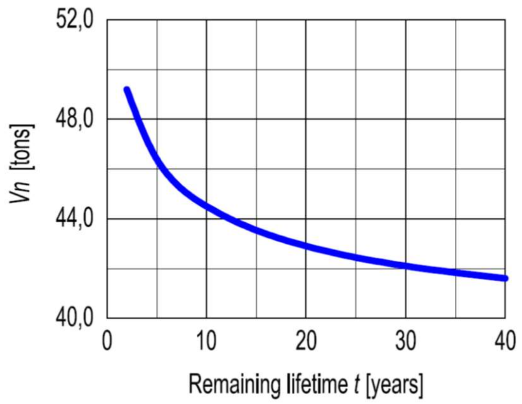

Figure 4: Normal load-bearing capacity for ULS vs. remaining lifetime (Full probabilistic method).

Table 4: Normal load-bearing capacity for ULS vs. remaining lifetime (Full probabilistic method).

Probability index (Remaining lifetime)	Normal load-bearing capacity V_n [tons]
$\beta_t = 2{,}828$ (2 years)	49,2
$\beta_t = 3{,}098$ (5 years)	46,4
$\beta_t = 3{,}279$ (10 years)	44,5
$\beta_t = 3{,}434$ (20 years)	42,9
$\beta_t = 3{,}512$ (30 years)	42,1
$\beta_t = 3{,}561$ (40 years)	41,6

Load-bearing capacity function *S* for the ultimate limit state was estimated based on 32 FEM simulations of superstructure failure assuming traffic load. The most appropriate type of the probability distribution functions and statistical characteristics were estimated and they are given in Table 5.

Table 5: Probabilistic model of normal load-bearing capacity of ULS.

Value		Unit	PDF*	Mean	CoV
Normal load-bearing capacity	*S*	[tons]	LN 3 par.	85,5	0,17

* LN 3 par. – 3-parametric lognormal distribution

4 Conclusion

In conclusion, it has been proved that probabilistic methods in combination with nonlinear FEM analysis represent an effective and practical tool in cases of evaluation of load-bearing capacity and reliability of existing structures. It has been demonstrated that probabilistic approach is less conservative and leads to slightly higher capacities than the deterministic one, which is mostly applied using current codes for design. However, for more detailed assessment of structures, more information is necessary to reduce model uncertainties and to reach the most accurate results. Therefore additional information from detailed diagnostic surveys should be used to improve the computational models. The adjustment of the target design value of reliability index in relation to the detailed diagnostic survey is also recommended.

Acknowledgement

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